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Fuzzy Colour Naming based on Sigmoid Membership Functions

Robert Benavente and Maria Vanrell
Computer Vision Center / Dept. Informàtica (UAB)
Bellaterra (Barcelona), Spain

Abstract

In this paper, we present some improvements towards a computational model for colour naming. Our model is based on fuzzy set theory and each colour category is considered a fuzzy set with a characteristic function. In previous works, we had proposed a model based on the use of a Sigmoid-Gaussian as membership function for the chromatic categories. Although it provided good results, the Sigmoid-Gaussian model presents some drawbacks due to the parameter dependence between the Sigmoid and the Gaussian functions. To overcome this, we propose two new functions which are based only on products of Sigmoids avoiding the problems introduced by the Gaussian function.

The results obtained by the new functions are compared to the previous ones. Although the improvement in terms of the fitting error is not very significant, the new functions show a higher degree of adaptability which will allow improving the modelling of the whole colour naming space. The functions are also used to label the Munsell colour array and the new membership functions provide similar categorizations than real observers.

Introduction

From the study of Berlin and Kay[1] about colour naming, a lot of work has been done about the topic from different points of view[2,3,4,5,6,7,8,9]. However, the computational automation of this visual task is still an open issue. Although in the last years some attempts have been made on this direction[10,11,12,13,14], there is still a lot of work to do. In this paper we present some improvements towards a computational model for colour naming.

Our model is based on fuzzy set theory. The formulation of colour categories as fuzzy sets was firstly proposed by Kay and McDaniel[2] and it provides a useful framework for the colour naming problem. According to the fuzzy set theory, each colour category, C_k, will be characterized by a function, f_{Ck}, which assigns to a colour sample $\underline{x}$, a value $f_{Ck}(\underline{x})$ in the [0,1] interval, and which represents the degree of membership of $\underline{x}$ to the colour term C_k or, in other words, the certainty that $\underline{x}$ will be named as C_k.

In previous works[13,14], we defined the set of properties that a function should fulfil to be a useful membership function for the fuzzy framework of colour naming. Moreover, a colour naming model for the eleven basic colour categories proposed by Berlin and Kay (white, black, red, green, yellow, blue, brown, purple, pink, orange and grey) was proposed. Such model was based on the combination of two well known functions: a Sigmoid and a Gaussian. Although the results were encouraging, the model presented some drawbacks.

Firstly, the Sigmoid-Gaussian used as membership function was not suitable for some of the categories (i.e. green and blue) due to the impossibility to adopt the form presented by these categories. Secondly, there were some regions of the colour space where the sum of the membership values for the eleven colour categories was not the unit, as it should be according to a fuzzy framework.

To overcome the problems of the Sigmoid-Gaussian model, in this work we propose two different membership functions based only on products of Sigmoids. Each one presents advantages for a part of the categories and the performance of both functions will be analysed. In this paper, the colour naming model is defined on the CIEL*a*b* colour space to take advantage of the perceptual uniformity of this space.

The rest of the paper is organized as follows. Firstly, we present a definition of all the colour naming models analysed in this work. Next, the fitting process of the membership functions for the different models is explained. In the results section, the performance of the two proposed models is compared to the previous results. Finally, the conclusions of the work are presented.

Colour Naming Models

In previous works[13,14], achromatic categories (white, black and grey) were modelled with multivariate Gaussian functions. For chromatic categories the study of the distribution of the membership values over the colour space, allowed us to define a set of necessary properties that membership functions should fulfil. These properties are:

- $f_{Ck}(\underline{x}) \in [0,1]$
- $f_{Ck}(\underline{x})$ has a plateau form in its central area that spans on a triangular basis with a principal vertex
- $f_{Ck}(\underline{x})$ must have parameters controlling the slope of the surface on the boundaries formed by the two sides of the triangular basis that share the principal vertex

- $f_{Ck}(\underline{x})$ must have parameters allowing asymmetry with respect to the central axis, that is, the bisector of the angle formed by the two sides of the triangular basis that have the principal vertex in common.

Our first approach, was the Sigmoid-Gaussian function, which is a product of two Sigmoids and a Gaussian. In this paper, we propose two functions which include a modification of the Sigmoid function and eliminate the product by the Gaussian to overcome the problems found with the Sigmoid-Gaussian model. In the next subsections the previous Sigmoid-Gaussian model and the two new functions are presented. In the three models, the CIEL*a*b* space is divided along the L dimension in three levels and for each intensity level, each one of the colour categories is modelled by a membership function.

Sigmoid-Gaussian

The Sigmoid-Gaussian function is defined as:

$$SG_{Ck}(a,b;\theta_1)=S_1(a,b;t_a,t_b,\alpha,\beta_a)\cdot S_1(b,-a;t_b,-t_a,\alpha,\beta_b)\cdot G(a,b;t_a,t_b,\alpha,\mu,\sigma) \quad (1)$$

where (a,b) are the coordinates of a point in the a*b* plane, $\theta_1=(t_a,t_b,\alpha,\beta_a,\beta_b,\mu,\sigma)$ and S_1 and G are given by:

$$S_1(a,b;t_a,t_b,\alpha,\beta)=\frac{1}{1+e^{-\beta(((a-t_a)\cos\alpha)-((b-t_b)\sin\alpha))}} \quad (2)$$

$$G(a,b;t_a,t_b,\alpha,\mu,\sigma)=e^{-\frac{1}{2}\left(\frac{\frac{a'-b'}{2^{1/2}}-\mu}{\sigma}\right)} \quad (3)$$

The meaning of the parameters is: t_a and t_b are a translation with respect to the center of the space, α is a rotation, β_a and β_b control the slopes of the plateau formed by the product of the two Sigmoid functions, μ is the mean and σ the variance of the Gaussian function. In equation (3), a' and b' are defined as:

$$a'=(a-t_a)\cos\alpha-(b-t_b)\sin\alpha \quad (4)$$

$$b'=(a-t_a)\sin\alpha+(b-t_b)\cos\alpha \quad (5)$$

The results obtained by the Sigmoid-Gaussian model in[14] were promising. However, some problems were detected on the model. The effect of the Gaussian, which is useful for some of the categories, is counterproductive for others such as green or blue. As the Gaussian is multiplying the product of the two Sigmoids, to obtain a wide plateau we need to have a high value of σ. Anyway, the plateau is never a perfect one since the Gaussian has a smoothing effect over the surface formed by the product of the two Sigmoids. Another drawback of the Sigmoid-Gaussian function is that it does not allow having membership functions covering a surface wider than an angle of 90º and this is what makes that the function can not model correctly the green category.

This lack of flexibility of the function makes that in some regions of the colour space the sum of the membership values for the eleven colour categories is not the unit, as it should be according to the fuzzy framework.

Double-Sigmoid

Our first proposed function, which we call Double-Sigmoid (DS_{Ck}) is a product of two Sigmoids and is given by:

$$DS_{Ck}(a,b;\theta_2)=S_1(a,b;t_a,t_b,\alpha_a,\beta_a)\, S_1(b,a;t_b,t_a,\alpha_b,\beta_b) \quad (6)$$

where $\theta_2=(t_a,t_b,\alpha_a,\alpha_b,\beta_a,\beta_b)$.

The parameters α_a and α_b are the angles that limit the wideness of the plateau as can be seen in figure 1. The rest of the parameters, t_a, t_b, β_a and β_b, have the same meaning as in the Sigmoid-Gaussian model.

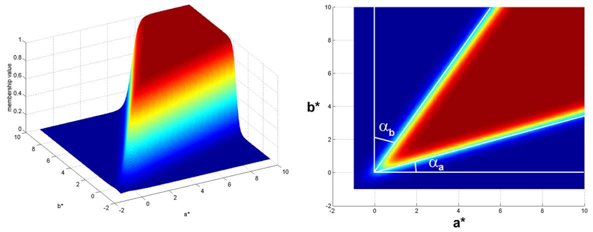

Figure 1.Example of the Double-Sigmoid function.

The fact that each Sigmoid of the function has a different rotation angle allows varying the wideness of the plateau. Hence, this new function solves one of the problems detected on the Sigmoid-Gaussian model. The effect of the two angles is similar to the product by the Gaussian function, but it does not have the smoothing effect of the Gaussian that did not allow wide plateaus in the previous model. Therefore, the new function is more flexible and allows a wider range of surfaces by just varying the parameters.

Triple-Sigmoid

The second function, which we will refer as Triple-Sigmoid (TS_{Ck}), is a modification of the above model and is given by:

$$TS_{Ck}(a,b;\theta_3)=S_1(a,b;t_a,t_b,\alpha_a,\beta_a)\cdot S_1(b,a;t_b,t_a,\alpha_b,\beta_b)\cdot S_2(a,b;t_a,t_b,\beta_d,r) \quad (7)$$

where $\theta_3=(t_a,t_b,\alpha_a,\alpha_b,\beta_a,\beta_b,\beta_d,r)$ and S_2 is given by:

$$S_2(a,b;t_a,t_b,\beta,r)=\frac{1}{1+e^{-\beta(((a-t_a)^2-r)-((b-t_b)^2-r))}} \quad (8)$$

t_a, t_b, α_a, α_b, β_a and β_b are the same parameters as in the Double-Sigmoid function. r is the radius of the S_2 function and β_d controls the slope of this third Sigmoid. The form of this function is showed in figure 2.

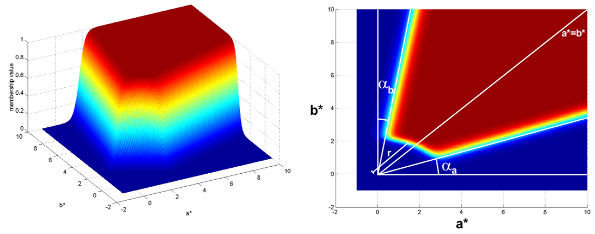

Figure 2. Example of the Triple-Sigmoid function.

The third Sigmoid added to the model is a circular hollow with radius *r*. The effect of this third Sigmoid allows improving the modelling of the central area of the colour space, that is the contact region between achromatic and chromatic categories.

Parameter Estimation

To estimate the parameters of the membership functions we used a set of colour naming judgements obtained from a fuzzy colour naming experiment performed to this purpose[15]. In this experiment, ten subjects were asked to distribute 10 points between the eleven basic colour terms for each sample presented. The scoring had to be done according to the certainty they had about the sample belonging to the different categories. Thus, if the subject was absolutely sure about the colour name of a sample, then the 10 points had to be assigned to the category corresponding to that name. Otherwise, if there was a doubt between two or more names, the 10 points had to be distributed between the categories corresponding to those names. Hence, for each sample, an eleven dimension colour descriptor CD($\underline{x}$) was obtained. The value of position *k* of the descriptor is the value given by the subject to the sample for the category C_k.

The experiment was developed under an illuminant with a CCT of 5955K and the set of samples that were used included 387 colour samples from the Munsell space. The experiment was done twice by each subject which implies a total number of 7740 observations. For each sample, the scores of the ten subjects were averaged and normalized to the [0,1] interval.

The results of the experiment were used as learning set during the fitting process which is a non-linear data-fitting problem in the least squares sense, that is, minimizing the mean squared error (MSE) between the membership values provided by the model and the values obtained in the experiment. For each of the 11 colour categories, the fitting was done using a Large-Scale optimization algorithm which minimizes the following expression:

$$\min_{\theta_k} \frac{1}{2} \sum_{j=1}^{s} \left(f_{C_k}(x^j, \theta_k) - CD_k(x^j) \right)^2 \qquad (9)$$

where s is the number of samples in the learning set, C_k is the colour category being modelled with k=1,...,11, x^j is the j-th sample of the learning set, $CD_k(x^j)$ is the k-th component of the colour descriptor obtained from the experiment for the sample x^j, and θ_k is the set of parameters of the membership function f_{Ck}.

Results

In this section, we present the results obtained by the two proposed membership functions in the modelling of the colour naming space. This results are compared with the results obtained by the Sigmoid-Gaussian model to observe the improvement provided by the two new Sigmoid-based functions.

To test the proposed functions the fitting error (MSE) on the psycho-physical data from the experiment (learning set) was computed. In addition, the samples from this set were assigned a colour name according to the highest membership value provided by the function tested, and this colour naming assignments were compared to the ones obtained in the experiment. Hence, the percentage of correctly labelled samples was also computed.

To show how the proposed membership functions improve the modelling of the whole colour space in terms of the fulfilment of the unit-sum fuzzy constraint (the membership values for a given point of the space must sum one) the colour space was sampled and the colour descriptors CD($\underline{x}$) for these points were computed. For each sampled point, its colour descriptor was added and the difference to one was computed. Table 1 summarizes these results.

Table 1. Results obtained by the three models (Sigmoid-Gaussian (SG), Double-Sigmoid (DS) and Triple-Sigmoid (TS)).

	Fitting error (MSE)	% correctly labelled samples	Mean distance to unit sum
SG	0.1541	86.05 %	0.3124
DS	0.0881	87.86 %	0.1507
TS	0.0840	90.70 %	0.1453

As can be seen in table 1, the two proposed functions reduce the fitting error on the learning set from 0.15 to 0.08. The improvement of the new functions can also be seen on the percentage of the correctly labelled samples of the learning set which increases from 86.05 % for the Sigmoid-Gaussian model to 87.86 % for the Double-Sigmoid and 90.70% for the Triple-Sigmoid. In the case of the mean distance to the unit sum, the results show a similar behaviour than the fitting error. The new functions reduce the mean distance to the unit sum from 0.31 to 0.15 and 0.14 which means that the unit-sum fuzzy constraint is better fulfilled by the new functions.

The new models have also been tested by categorizing the Munsell colour array. The categorization obtained has been compared to previous results. In figures 3 and 4 two categorizations provided by real human observers are presented. The first one (figure 3) is the result of Berlin and Kay[1] experiments for English language. The second is the categorization made by a 35-

year-old English male[16]. Next we present the categorizations obtained by our three membership functions: Sigmoid-Gaussian (figure 5), Double-Sigmoid (figure 6) and Triple-Sigmoid (figure 7).

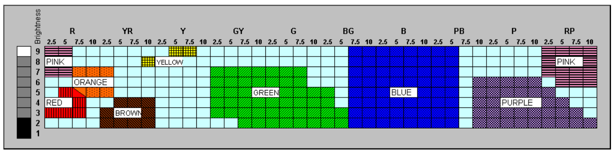

Figure 3.Munsell categorization from Berlin and Kay results for English.

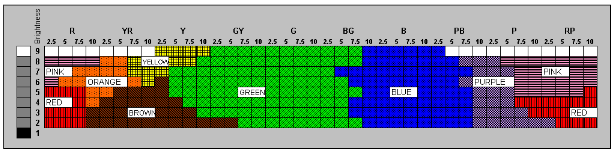

Figure 4.Munsell categorization made by a 35-year-old English male.

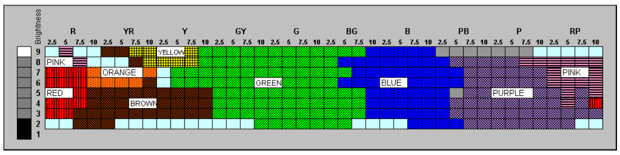

Figure 5.Munsell categorization obtained by the Sigmoid-Gaussian model.

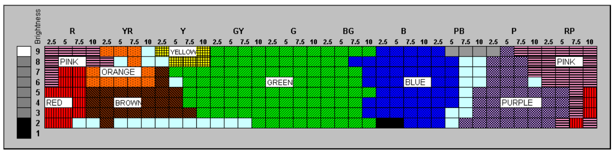

Figure 6. Munsell categorization obtained by the Double-Sigmoid model.

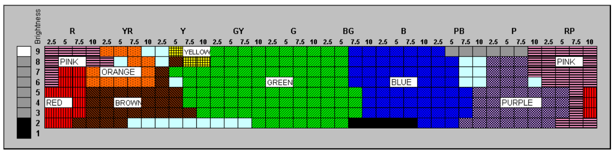

Figure 7. Munsell categorization obtained by the Triple-Sigmoid model.

In the figures, the samples from the Munsell array that had a highest membership value lower than 0.5 were ignored. These samples are represented in a light colour and do not have any pattern in the figures since they do not have a colour name assigned.

As can be seen in the figures, the new membership functions provide similar categorizations than real observers. Although there are some errors in these categorizations, the regions covered by the different categories coincide quite accurately with the region proposed by Berlin and Kay for English or by the real observer (figure 4).

Conclusion

In this paper we have proposed two new membership functions for fuzzy colour naming. The functions have been defined and tested to show the improvements that the functions obtain when are compared with the previous Sigmoid-Gaussian model.

Although the improvement in terms of the fitting error is not high, the important is that the new functions are easily adaptable to the learning data set obtained from the psycho-physical experiment.

These results, obtained on the CIEL*a*b colour space, could be easily exported to other colour spaces as we had done with the previous Sigmoid-Gaussian model[14]. This is very encouraging since it will allow us to propose our model as a useful computational model for computer vision applications by just computing the model parameters on the device space of choice. Although colour naming has not been widely used in computer vision, it can be a very useful visual cue for applications such as image segmentation, image indexing and retrieval, and robotics.

Acknowledgements

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Biography

Robert Benavente is a research fellow at the Computer Science Department of the Universitat Autònoma de Barcelona (UAB) and is affiliated to the Computer Vision Center. He received his MSc degree in Computer Vision from the UAB in 2000. He is currently pursuing his PhD in Computer Science. In addition, he participates in the development of industrial applications related to quality control processes based on computer vision. His research interests include colour perception and representation, colour naming and machine vision.

Maria Vanrell joined the Computer Science Department of the Universitat Autònoma de Barcelona (UAB) in 1990 and became associated professor in 1997. Currently, she is also a member of the research staff of the Computer Vision Center (CVC). M. Vanrell is an active member of the Image Analysis and Pattern Recognition Spanish Association (AERFAI), a branch of the IAPR. Since 1996, when she received her PhD at the UAB for her work on Computational Texture Representation, her research interests are: texture perception and representation, computational colour representation, colour naming and the study of colour texture interactions. She has been the head and coordinator of several research and industrial projects and has published several papers in national and international conferences and journals.